# Next generation aircraft design considering airline operations and economics

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**Traditional approaches to design and optimization of a new system often use a system-centric objective and do not take into consideration how the operator will use this new system alongside other existing systems. When the new system design is incorporated into the broader group of systems, the performance of the operator-level objective can be sub-optimal due to the unmodeled interaction between the new system and the other systems. Among the few available references that describe attempts to address this disconnect, most follow an MDO-motivated sequential decomposition approach of first designing a very good system and then providing this system to the operator who, decides the best way to use this new system along with the existing systems. This paper addresses this issue by including aircraft design, airline operations, and revenue management "subspaces"; and presents an approach that could simultaneously solve these subspaces posed as a monolithic optimization problem rather than the traditional approach described above. The monolithic approach makes the problem an expensive Mixed Integer Non-Linear Programming problem, which are extremely difficult to solve. To address the problem, we use a recently developed optimization framework that simultaneously solves the subspaces to capture the "synergy" in the problem that the previous decomposition approaches did not exploit, addresses mixed-integer/discrete type design variables in an efficient manner, and accounts for computationally expensive analysis tools. This approach solves an 11-route airline network problem consisting of 94 decision variables including 33 integer and 61 continuous type variables. Simultaneously solving the subspaces leads to significant improvement in the fleet-level objective of the airline when compared to the previously developed sequential subspace decomposition approach.**

## Nomenclature

| | | |
|---|---|---|
| $BH_{a,j}$ | = | Block hours of aircraft type $a$ on route $j$ |
| $dem_j$ | = | Daily passenger demand on route $j$ |
| $EI$ | = | Expected Improvement |
| $fleet_a$ | = | Number of aircraft type $a$ |
| $k_I$ | = | Number of integer type design variables of the problem |
| $MH_{a,j}$ | = | Maintenance hour per block hour of aircraft type $a$ on route $j$ |
| $pax_{a,j}$ | = | Number of passengers per flight on aircraft type $a$ on route $j$ |

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| | | |
|---|---|---|
| $priceX_t$ | = | Non-restrictive fare class on time window $t$ |
| $priceY_t$ | = | Restrictive fare class on time window $t$ |
| $trip_{a,j}$ | = | Number of trips by aircraft type $a$ on route $j$ |
| $\mathbf{x_C}$ | = | Vector of continuous type design variables |
| $\mathbf{x_I}$ | = | Vector of integer type design variables |
| $\mathbf{x}_{()}{}^{lb}, x_{()}^{ub}$ | = | Design variable bounds |
| $z_{1,j}$ | = | Booking limit on time window 1 on route $j$ |

# I. Introduction & Motivation

A typical aircraft design process involves designing an aircraft for a particular design range and a payload capacity, or perhaps a set of payload-range combinations. An aircraft sizing analysis tool can size the aircraft (i.e., predict the aircraft size, weight, performance, cost, etc.) to meet the desired design mission range and seat capacity requirements. In reality, however, aircraft fly missions of different ranges with varying payloads to meet the needs of the operators. For example, an airline may allocate the new aircraft on short routes to meet the high passenger demands on those routes. When the sizing process uses a single design range and payload, the resulting aircraft is not optimal for routes other than its defined payload-range combination. These sub-optimal "off-design" operations lead to increased operating costs (and likely reduced profit) on these short routes for the airlines. A profit-seeking airline would like to maximize the profit across the entire network it operates, thereby raising the need to formulate an optimization problem that accounts for design together with the operational and economic behavior of the airline. When an airline ponders acquisition of a new aircraft to operate alongside its existing fleet, that new aircraft should work to maximize the profit of the airline. The framework presented in this paper seeks to describe a new aircraft design approach that directly influences the airline level objective taking into consideration the operations and economics of the airline as decision variables into the optimization problem formulation. The figure below shows how the design, operations and economics "subspaces" interact, demonstrating the need for the monolithic formulation that would improve the fleet-level objective of the airline.

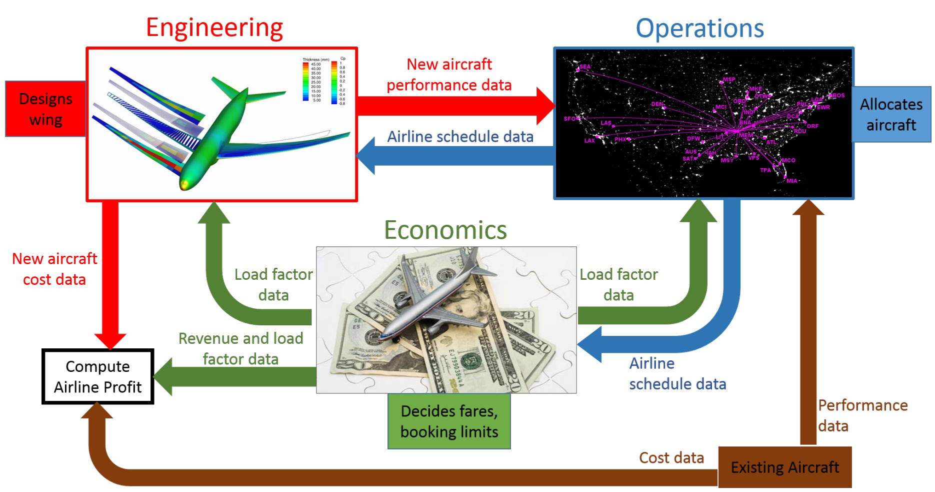


**Fig. 1 Overview of the design, allocation and the revenue management problem and the interactions between them**

The engineering system shown in Fig 1, also referred to in this paper as the aircraft design optimization subspace, designs the wing geometry and the engine size, and then flies the aircraft on routes decided by the operation subspace with the number of passengers onboard driven by the economic subspace. Similarly, the operations subspace uses the new aircraft performance data along with the existing aircraft performance data and merges the load factor information from the economics subspace to allocate aircraft on different routes. The economic subspace receives the total available capacity (seats) on a given route from the operation subspace and seeks to sell these seats to customers by setting the fare values. Finally, the fleet-level profit of the airline is computed using the cost and the revenue information. The approach maximizes the airline profit by addressing all of these subspaces simultaneously.

Individually, these subspaces may be solved in sequence, posing them as separate optimization problems as demonstrated in the initial and previous efforts to address this type of problem [1–3]. Solving the problem in sequence makes it computationally tractable because this sequential decomposition breaks down the large MINLP problem into smaller subproblems. However, doing so omits some of the interactions between the subspaces (see Fig. 1) that could lead to sub-optimal solutions with respect to the fleet-level profit. In this sequential approach, at the time of designing the new aircraft, no information exists about how the airline allocates this new aircraft, or how many passengers fly aboard this new aircraft on these allocated routes. This subspace decomposition strategy does not exploit the synergy that exists between the subspaces. As an example, solving the subspaces sequentially first involves the aircraft design optimization, which is often posed as a Non-Linear Programming (NLP) problem. The design variables associated with the aircraft design process are typically continuous variables that define the wing and the engine parameters, and the aircraft design is subjected to various performance constraints. Further, depending on the level of fidelity desired in the various disciplines of the aircraft design subspace (e.g., aerodynamics, structures, control, propulsion, etc.), this problem may become computationally very expensive. On the other hand, the airline allocation subspace as a simplistic representation of a typical airline operation is often posed as an Integer Linear Programming (ILP) problem. The integer design variables of the allocation problem are the number of trips made by each aircraft type on each route. This problem is the analog of setting the schedule for the airline by determining the number of trips each aircraft type in the airline's fleet makes on each route. Once the schedule has been set, the airline tries to sell the available seats to the passengers at fares decided by the economic (revenue management) subspace. The revenue management subspace decides the ticket price for advance purchase of a fare class and the booking limit imposed on that fare class with the objective of maximizing the revenue on that route. The booking limit protects a number of seats for the subset of passengers who are willing to pay a higher price for last-minute booking. Deciding the right price for a fare class and the corresponding booking limit is in itself an optimization problem, which is often solved using Nonlinear Programming (NLP), where the continuous type design variables are the ticket prices and the booking limits.

In this paper we present an approach to solve the challenge problem: design-allocation-revenue management (DARM) posed as a single monolithic optimization problem that captures the interactions between the subspaces (also appears in Ref. [4]). Combining these subspaces makes the problem a Mixed Integer Non-Linear Programming (MINLP) problem and is very hard to solve, if not impossible with available solvers. With recent advancement in the field of operation research, newer algorithms can solve MINLP problems [5], but their application to simultaneously solving the design, allocation, and revenue management problem is limited by the computationally expensive and sophisticated aero-structure-propulsion modules used by the aircraft design companies.

To address this challenge problem, we use the recently developed optimization framework AMIEGO [6] available as a driver within NASA's OpenMDAO [7]. AMIEGO employs a hybrid approach combining features from several different optimization algorithms. The optimization framework leverages the efficiency of a gradient-based optimizer to exploit the large-scale continuous design space together with the efficient global exploration capability of EGO [8] to explore the integer design space. In doing so, it combines the benefits that these individual optimization approaches bring to the table, i.e., the ability to handle a very large-scale design optimization problem in the continuous space with expensive analysis models using a gradient-based approach, while globally exploring the integer design space using the EGO framework, and limiting the expensive continuous optimization runs to as few as possible. Addressing a large-scale integer design space is still problematic, but the framework combines a kriging surrogate model with Partial Least Squares (PLS) to mitigate this issue.

## II. Literature Review

There exists plethora of literature that has advanced the state of the individual subspaces. However, to the best of author's knowledge, no literature exists that has posed and solved the design, allocation, and revenue management as a monolithic optimization problem. Most of the early and recent studies that combine the aircraft design and allocation subspaces have done so within the context of system-of-systems[1–3, 9–11]. One of the first steps was the MDO-motivated decomposition approach by Mane et al. [1], where they decomposed the large MINLP problem into two smaller subspace problems, with a top level driver that decides on the design range and seating capacity of the new "yet-to-be-developed" aircraft, as appears in Fig. 2.

The aircraft design subspace problem sizes the new, "yet-to-be-developed" aircraft for a specific design mission range and payload (which comes from the top level) via an NLP problem formulation. The newly designed aircraft along with the existing aircraft in the fleet are allocated to the route network via an MILP problem formulation that maximizes the airline-level profit. This profit value then becomes the solution to the top-level problem. The top-level

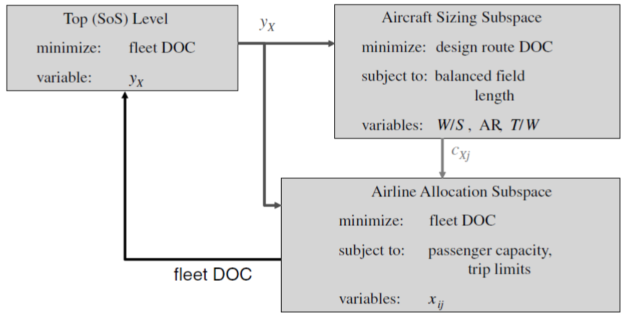


**Fig. 2 Sequential decomposition approach (adapted from Ref. [1])**

subspace then generates a new set of requirements for the “yet-to-be-developed” aircraft (a new seat-range combination) and re-solves the aircraft design optimization and airline allocation problem sequentially.

This sequential subspace decomposition approach gets around the challenges and hurdles of an MINLP problem and solves two relatively easier subproblems –an NLP problem (design) and an MILP problem (allocation). This sequential approach reduces the computational cost and can solve a small-scale aircraft design optimization problem and a moderate-scale airline allocation problem leveraging the Linear Programming (LP) formulation of the allocation subspace. This approach has been applied to include multiple airlines, where the new yet-to-be-developed aircraft is designed to serve two airlines [12]. A follow-up effort using the same subspace decomposition framework addresses uncertainties in the design and the allocation subspace for military and commercial application [13, 14]. Although a significant amount of work has been done using the decomposition framework, the subspaces are inherently coupled and the subspace decomposition approach could not capture the synergy that exists between the various subspaces.

To address this shortcoming of the subspace decomposition approach, we investigate if there is an alternate way to approach this problem that would capture the interactions between the subspaces, leading to a better fleet-level solution. As a starting step, the effort focused on establishing the interactions between the aircraft design and the airline allocation subspace while ignoring the top-level driver. We assume, as though through some prior analysis, that the seating capacity and the design range of the “yet-to-be-developed” aircraft is known and seeks to solve the design-allocation subspaces posed as a monolithic optimization problem.

The first attempt to demonstrate the combined problem uses a form of Branch and Bound (BB) algorithm that enforces the integer constraints and solves only the mission-allocation problem (i.e., it ignores the aircraft design) for a simplistic three-route airline network problem [15]. For this three-route example problem, the simultaneous approach showed an improvement in the airline profit compared to the sequential approach. However, the BB had issues converging to a local solution without global exploration. Further, the problem might get computationally very expensive with high-fidelity aircraft design and an increased size of the airline network, as observed earlier by Mane at al. [1], thereby establishing the need for a better approach. Follow-up efforts [16, 17] extended the three-route allocation-mission problem to consider a more realistic 128-route network and leverage a parallel computing framework. The work employs a high-fidelity aerostructural analysis tool for the large airline network problem, has over 6000 design variables, and uses the adjoint-based method to compute the analytic derivatives. This work demonstrates the benefits of simultaneous optimization of the design-allocation subspace but makes a major assumption in the process because it relaxes the integer constraints on the allocation variables. Relaxing the integer constraints on the problem makes it a classic continuous NLP problem and is solved using a gradient-based approach. This relaxing of the integer variables yields no practical meaning to the number of trips made by an aircraft type. One could round-off the continuous solution to the nearest integer value to address the integrality constraints; however, that may lead to infeasible or sub-optimal solutions [18]. This motivated us to investigate the development of an MINLP solver that can handle computationally expensive analysis tools, support mixed-integer type design variables, and solve the combined high-fidelity aircraft design optimization and airline allocation problem.

With the inclusion of the integer type design variables, the problem becomes challenging to solve. Roy and Crossley[19] proposed a new MINLP approach to address this kind of problem. Keeping in mind the expensive nature of the high-fidelity aircraft design optimization, the approach uses an Efficient Global Optimization (EGO)-like framework to handle the integer and continuous type design variables using different optimizers. The framework leverages the global exploration capability of EGO to optimize the integer design space and uses the capability of gradient-based methods to efficiently explore the large-scale continuous design space. The paper demonstrates a small-scale aircraft design-allocation problem posed as an MINLP problem and solved using the EGO-like approach. The study used a simplistic Raymer-based low-fidelity aircraft sizing tool to size the aircraft and obtain the aircraft performance data [20]. The result using the EGO-like framework shows the simultaneous optimization yields an aircraft design solution that leads to slightly better fleet-level profit compared to the sequential decomposition approach for the three-route problem. This motivates us to explore further in this direction of research with an added airline economic model that now includes a revenue management subspace solved within the OpenMDAO framework.

## III. Methodology

### A. Solving the problem

As mentioned earlier, the combined design-allocation-revenue management is an MINLP problem and is very difficult to solve given the presence of both integer and continuous type design variables and expensive and sophisticated analysis tools. The work here uses the recently developed EGO-like optimization algorithm AMIEGO, named as A Mixed Integer Efficient Global Optimization [6], to solve the aircraft design-allocation-revenue management problem in a simultaneous approach. Preliminary results demonstrate [4, 6] that AMIEGO has been successful in addressing several test problems of varying difficulty levels. An overview of the AMIEGO algorithm appears in Fig.3 and a brief description of each step as outlined in Ref. [4, 6] follows below. The red blocks use the EGO framework that explores the integer design space, while the blue block leverages the use of a gradient-based approach to explore the large scale continuous design space.

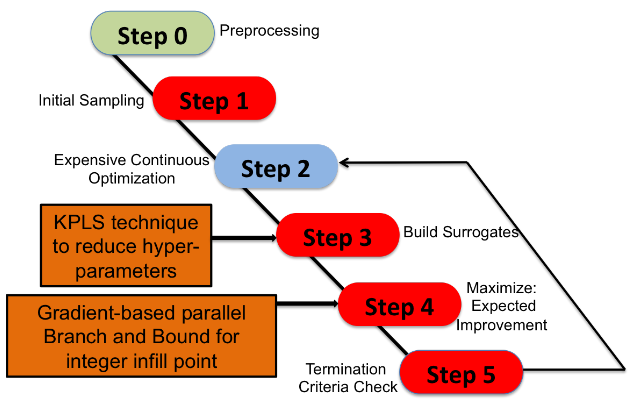


**Fig. 3 An overview of AMIEGO optimization framework**

**Step 0:** Separate the integer type design variables from the continuous type design variables of the original MINLP problem.

**Step 1:** Generate a set of initial integer points $\mathbf{x_I^0}$. Any Design of Experiment (DOE) methods may be used to generate the set of starting points rounded to the nearest integer point.

**Step 2:** Perform the continuous optimization. This step is different from the traditional EGO algorithm or any related work on Mixed Integer Surrogate Optimization. Instead of evaluating the objective/ constraints, the step performs a complete optimization with respect to the continuous design variables for each integer point using any gradient-based optimizer. All the continuous type design variables of the original problem appear in this step. The integer points from Step 1 are supplied as parameters and stay constant during the optimization. The subproblem formulation of this step appears below.

$$\begin{aligned} \text{Given: } \mathbf{x_I}, \quad \underset{\mathbf{x_C}}{\text{Minimize:}} \ & f(\mathbf{x_C}, \mathbf{x_I}) \\ \text{Subject to: } & g_1(\mathbf{x_C}, \mathbf{x_I}) \leq 0 \\ & \mathbf{x_C}^{lb} \leq \mathbf{x_C} \leq \mathbf{x_C}^{ub} \end{aligned} \tag{1}$$

It is important to state that this optimization is still local with respect to the continuous design variables and is sensitive to the starting point as required by any gradient-based approach. For our design-allocation-revenue management problem, we start-off with data from an existing aircraft that we envision to model as a future aircraft. As an example, we may use an existing B777-200 aircraft as our starting point with the idea to obtain a better futuristic version of a similar class of aircraft.

**Step 3:** Build using the result of the continuous optimization obtained in the previous step. We are using Kriging for training the surrogate. This surrogate model, as a function of the integer type design variables $\mathbf{x_I}$ is trained using the integer points at which the objective function is minimized with respect to the continuous design variables in Step 2. This is the surrogate of the already minimized objective function, but minimized with respect to the continuous design variables.

AMIEGO also combines kriging with the Partial Least Square (PLS) regression [21]. Combining Kriging with PLS regression, it is possible to significantly reduce the number of hyper-parameters i.e., the design-variables in the surrogate training optimization step. The method constructs a new covariance kernel function with reduced number of hyper-parameters based on the information provided by the PLS technique.

**Step 4:** Find an integer solution to the problem of maximizing the constrained expected improvement function. This auxiliary optimization problem to maximize the expected improvement function is also only with respect to the integer type design variables of the original problem. As opposed to finding a continuous solution like in traditional EGO algorithm [8], this step finds a solution that also satisfies the integer constraints. This integer solution in turn, is the solution to the integer design space and is the new infill point that appear as parameter to the continuous optimization in Step 2.

Several optimization algorithms have difficulties maximizing the expected improvement ($EI$) function, due to its non-convexity nature. Fewer algorithms are effective in finding the near global solution to this sub-problem. Most examples in the literature use evolutionary-based algorithms like Genetic Algorithm (GA) to maximize the expected improvement function. Further we seek to obtain an integer solution which makes this sub-problem a pure Integer Non-Linear Programming (INLP) problem. However, this auxiliary INLP problem is computationally cheap to solve and after rigorous mathematical simplifications, it is possible to obtain the gradient and Hessian information of the objective 'Expected Improvement' function; this information can be exploited by a mathematical programming solver for a global solution [8]. AMIEGO uses a newly developed gradient-based Branch-and-Bound (BB) approach available within OpenMDAO to solve this $EI$ maximization problem [4, 6]. However, the BB, slows down drastically as the number of integer variables of the problem grow. This is because, given the highly nonlinear nature of $EI$ design space, it is really difficult for a gradient-based approach to obtain a valid lower bound via a local search. We use a simple Genetic Algorithm [22] on top of BB as a heuristic to obtain the valid lower bound needed for this $EI$ maximization problem. Further, combining GA with the BB provides a more robust solution (at the expense of marginal increase in computational time) than a pure GA-based approach typically followed in EGO algorithms.

Unlike the approach presented by Roy et al. [4, 6] to handle the constraints, the current implementation of the AMIEGO framework uses an exterior penalty approach in which a penalty term is formed by multiplying the violation by some factor. This penalty term is directly added to the objective function value obtained from Step 2. This in turn helps reduce the computational burden of building the surrogates and obtaining a valid upper bound of all the constraint functions as needed by the MINLP BB algorithm. However, this penalty approach requires a penalty multiplier factor that penalizes any infeasible design solution coming from the continuous optimization in Step 2. The current implementation of AMIEGO uses a penalty multiplier factor of 10 which seems appropriate for our application problem. However, we would like to use a generalized penalty multiplier factor value and that would require additional research to obtain the right penalty multiplier factor for any given problem.

**Step 5:** Terminate the algorithm when the expected improvement value falls below a certain percentage of the best known feasible solution. The current implementation of the framework uses a tolerance limit of 0.1% of the best found solution.

### B. Problem description and formulation

The ultimate goal is to design an advanced version of a passenger aircraft with 162 seats and a design range of 2940 nmi that is yet-to-be-acquired by a hypothetical airline that has a defined route network and demand structure. The seating capacity and design range of this new aircraft is based upon that of the Boeing 737-800; the popularity of this aircraft on current US domestic routes led to this choice. Our approach captures the operational and economic characteristics of the airline via a simple allocation model and a revenue management model, respectively, and treats aircraft design, airline allocation, and revenue management as three separate disciplinary subspaces that we seek to optimize simultaneously.

Figure 4 shows the 11-route structure of the hypothetical airline network. The network represents as hub at Boston connecting the eleven cities on the spokes. Table 1 shows the distance of each route from the hub in nautical miles and the corresponding one-way daily demand. For this problem, the airline is assumed to have two different types of existing aircraft already in operation. These include 10 aircraft of B757-200 type and 14 aircraft of A320-200 type. Also, the airline seeks to buy five of the new "yet-to-be-deployed" aircraft.

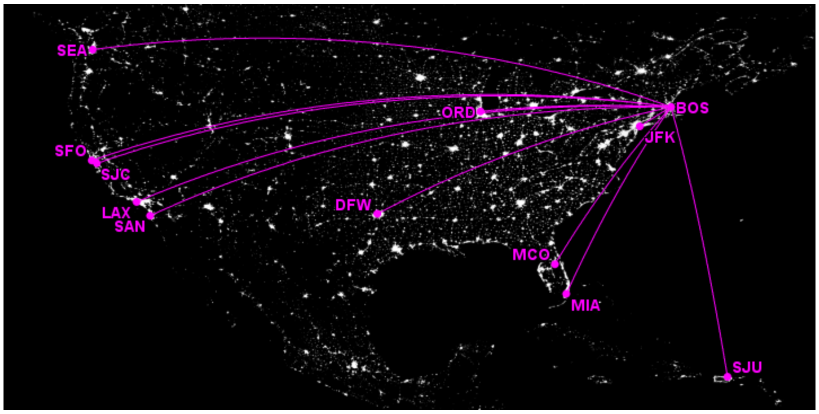


**Fig. 4 Eleven route airline network with hub at Boston**

**Table 1 Table showing destination city from the hub at Boston (BOS), the route distance, and the daily demand (one-way).**

| Route number | Origin-destination | Destination City | Distance (nmi) | Daily demand |
|---|---|---|---|---|
| 1 | BOS-JKF | New York, NY (JFK) | 162 | 41 |
| 2 | BOS-ORD | Chicago, IL (ORD) | 753 | 1009 |
| 3 | BOS-MCO | Orlando, FL (MCO) | 974 | 89 |
| 4 | BOS-MIA | Miami, FL (MIA) | 1094 | 661 |
| 5 | BOS-DFW | Dallas, TX (DFW) | 1357 | 1041 |
| 6 | BOS-SJU | San Juan, Puerto Rico (SJU) | 1455 | 358 |
| 7 | BOS-SEA | Seattle, WA (SEA) | 2169 | 146 |
| 8 | BOS-SAN | San Diego, CA (SAN) | 2249 | 97 |
| 9 | BOS-LAX | Los Angeles, CA (LAX) | 2269 | 447 |
| 10 | BOS-SJC | San Jose, CA (SJC) | 2337 | 194 |
| 11 | BOS-SFO | San Francisco, CA (SFO) | 2350 | 263 |

The MINLP problem formulation to simultaneously solve the aircraft design-allocation-revenue management (DARM) problem appears in Table 2.

We seek to maximize the profit of the airline. The aircraft design variables to size the new aircraft include those that define the wing geometry and the engine performance. For this paper, we use FLOPS [23] as the sizing tool to model the next generation aircraft. Later, as a part of future work, we will switch to a more sophisticated physics-based high fidelity tool [17] as a part of the aircraft sizing process. For this application, the airline uses the following allocation and revenue management formulation.

**Airline Allocation Formulation:** Given a fleet of aircraft and their cost and performance data, the airline allocates them across its routes in the network to meet the daily demand. The design variables involved in the allocation problem are the number of trips $trip_{a,j}$ made by aircraft type $a$ on route $j$. These are integer type design variables. This subproblem is subject to two types of operational constraints. The first is the demand constraint, mathematically

**Table 2 Problem formulation for the fully coupled MINLP design-allocation-revenue management problem.**

| | | |
|---|---|---|
| Maximize | Fleet-level profit of the airline | |
| | (variable description) | (variable type) |
| With respect to | wing shape variables | continuous |
| | engine thrust | continuous |
| | flights per day | integer |
| | fare class prices per route | continuous |
| | booking limit per route | continuous |
| Subject to | aircraft performance constraints | |
| | demand constraints | |
| | aircraft count constraints | |

represented as,

$$\sum_{a=1}^{A}(trip_{a,j} \cdot pax_{a,j}) \leq dem_j, \forall j = 1, .., J \tag{2}$$

where, $trip_{a,j}$,$pax_{a,j}$ represents the total number of trips made by aircraft type $a$ on route $j$ and the total number of passengers carried per trip by aircraft type $a$ on route $j$ respectively. The variable $dem_j$ is the maximum demand on route $j$ (see Table 1). This constraint ensures the total number of passengers carried on a given route never exceeds the maximum demand on that route. The variable $pax_{a,j}$ comes from the revenue management subspace for the simultaneous approach. The second is the aircraft count constraint, which can be expressed as

$$2\sum_{j=1}^{J} trip_{a,j} \cdot [BH_{a,j}(pax_{a,j}) + MH_{a,j} + 1] \leq 24 \cdot fleet_a, \forall a = 1, .., A \tag{3}$$

This constraint ensures the total number of aircraft of a particular type utilized within a 24 hours time frame never exceeds the maximum number of available aircraft type. Note, the block hour $BH_{a,j}$ is a function of the load factor, which is also decided by the revenue management subspace.

**Revenue Management Formulation:** On a given route, the airline offers two classes of fare: $priceX$ and $priceY$. The fare class $priceX$ is more expensive, and hence, it is less restrictive than the fare class $priceY$. Each airline also sells these two fare classes across two different time windows. Time window 1 seeks to capture the leisure travelers and requires advance purchasing. Time window 2 is basically for the last minute business travelers, who are less flexible with their schedule but are willing to pay a higher price. The revenue management also needs to decide on the booking limit for fares during time window one and to protect seats for the fares offered in time window two. This version of the revenue management strategy is similar to the one proposed by Cizaire and Belobaba [24]. However, the work here extends the concept at a network level of the airline as opposed to a single point-to-point flight as presented by Cizaire and Belobaba [24] and ignores the uncertain nature of the passenger demand. The figure below shows an example revenue managing strategy with two fare classes and two booking limit periods.

To pose this as an optimization problem, the decision variables are $priceX_{t,j}$, $priceY_{t,j}$ and $z_{1,j}$. The variables $priceX_{t,j}$, $priceY_{t,j}$ are the non-restrictive and restrictive class fares respectively for the time window $t$ on the route $j$ for the airline. The variable $z_{1,j}$ is the booking limit imposed on the number of seats available for purchase in the time window one on route $j$ by the airline. The remaining seats are protected for the two classes of fares during the second time window. This subproblem is essentially an unconstrained Non-Linear Programming (NLP) problem; however, when integrated with the allocation subspace, the total accepted booking across both the time windows is constrained to be less than or equal to the demand of the airline on that route.

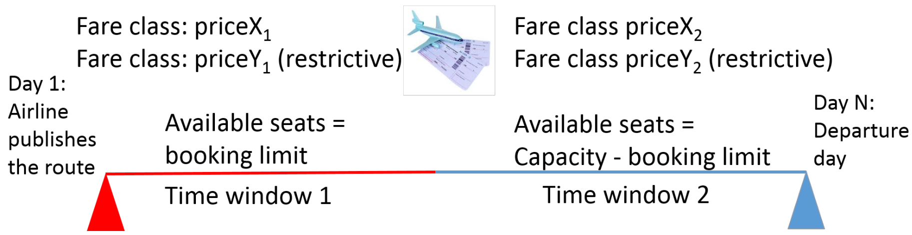


**Fig. 5 Revenue management problem**

## IV. Results

The allocation-revenue management only approach (aircraft design variables fixed to a B737-800 model) found solutions to the "baseline" scenario, while the sequential approach and AMIEGO find solutions to the DARM problem. In each of these cases, the airline-level goal is to maximize profit while satisfying the constraints.

The figure below shows the normalized airline profit* for the solutions of the three scenarios. The simultaneous

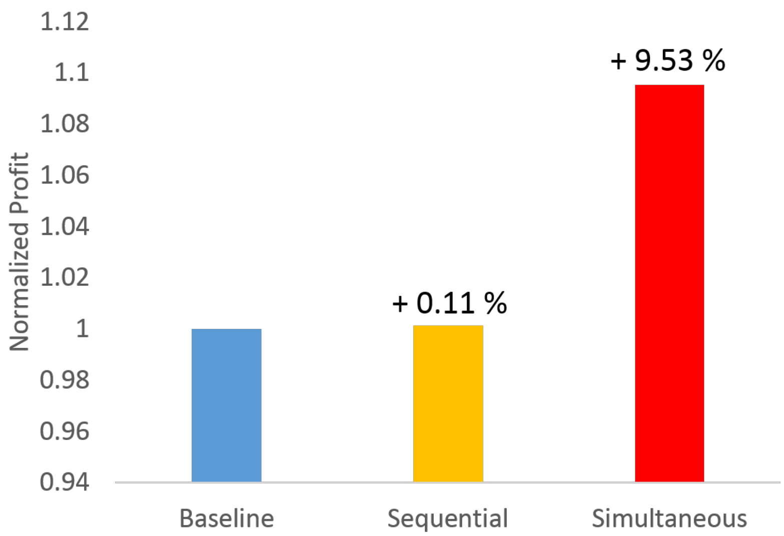


**Fig. 6 Airline profit for the three scenarios**

approach shows a 9.53% improvement in the fleet-level profit as compared to the baseline scenario, whereas the sequential approach shows only a marginal improvement in the airline profit (0.11% increase). The difference in the airline profit between the simultaneous and the sequential approach is about 9.42%.

Starting with the aircraft sizing subspace, Table 3 below shows the difference in the wing plan form for the three scenarios. The sequential approach yields a higher aspect ratio wing and a reduced swept wing design solution. On the other hand, the simultaneous approach yields a design solution with a reduced wing area and a slightly reduced thrust per engine.

Table 3 also shows the performance data of the "new" aircraft at the design mission range of 2940 nmi for the three scenarios. The aircraft is sized for a 2940 nmi design mission range with 162 seats onboard. The results in the table show that the sequential approach yields a lighter gross weight aircraft, which would generally indicate a better performing aircraft for this design mission range. The baseline scenario yields an aircraft with highest maximum take-off weight and the simultaneous scenario result lies somewhere in between, in gross weight. These results are intuitive, because the sequential approach solves the aircraft design problem to minimize the aircraft total operating cost using the design mission range and payload, so the aircraft might be expected to perform the best on this payload range

*The normalized profit for the simultaneous case is smaller than what is presented in Ref. [4]. This is probably due to the penalty approach upgrade in AMIEGO as compared to the expected violation approach to handle the constraints in Step 4. Identifying the right penalty multiplier is a difficult task and requires additional research.

**Table 3 Aircraft design variables for the three scenarios.**

| | Baseline | Sequential | Simultaneous |
|---|---|---|---|
| **Design variables** | | | |
| Aspect ratio | 9.4 | 11.99 | 9.34 |
| Taper ratio | 0.159 | 0.1 | 0.157 |
| Thickness-to-chord ratio | 0.1338 | 0.091 | 0.1336 |
| Wing Area [sq.ft] | 1345.5 | 1343.3 | 1191.3 |
| Sweep [deg] | 25.0 | 13.98 | 25.16 |
| Thrust per engine [lbs] | 24200.0 | 24200.0 | 24118.4 |
| **Performance data** | | | |
| (sized for design mission range of 2940 nmi) | | | |
| Gross weight [lbs.] | 169,028 | 163,612 | 166,680 |
| Total operating cost [$] | 67,418 | 66,449 | 66,998 |
| Empty weight [lbs.] | 79,593 | 83,272 | 77,866 |
| Fuel burn [lbs.] | 46,967 | 37,884 | 46,357 |
| Thrust to weight ratio | 0.286 | 0.296 | 0.289 |
| Wing loading [lbs./sq.ft.] | 125.6 | 121.8 | 139.9 |

combination. However, when assessed from the airline operations perspective, this aircraft leads to a lower airline profit value than when AMIEGO solves the simultaneous scenario problem. From an acquisition practitioner standpoint, the airline (operator) may not care much about the values of the design variables for the new aircraft; however, they may care more about whether the new aircraft maximizes the airline's profit. A detailed discussion of how this new aircraft could benefit the airline is discussed later.

Figure 7 shows the net profit per day[†] (one-way) on each route for the three scenarios. There is some improvement

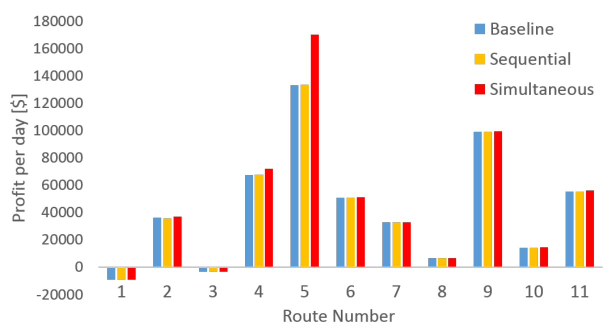


**Fig. 7 Profit on each route for the three scenarios.**

in profit on routes 2, 4 and 5 for the simultaneous scenario. These are the routes with high demands (see Table. 1). For the remaining routes, all the three scenarios yield similar profit values. Figure 8 shows the allocation results. The new aircraft gets allocated on route 2 and 4 for all the three scenarios (blue bars). For the baseline case there is no new aircraft; instead, it has five B737-800-like aircraft as represented by the blue bars in the figure. Overall, the simultaneous approach makes fewer trips per day on route 4 and 5 as compared to the baseline and the sequential scenarios, thereby a lower total revenue, but also at a reduced total operating cost. This increases the net profit. Intuitively, one may think flying more passengers directly yields more profit, but this is not true in this case. Flying more flights (in baseline and

[†]The price-demand elasticity model uses an average ticket price data obtained using FLEET tool [25] and cost is generated using FLOPS [23] cost model

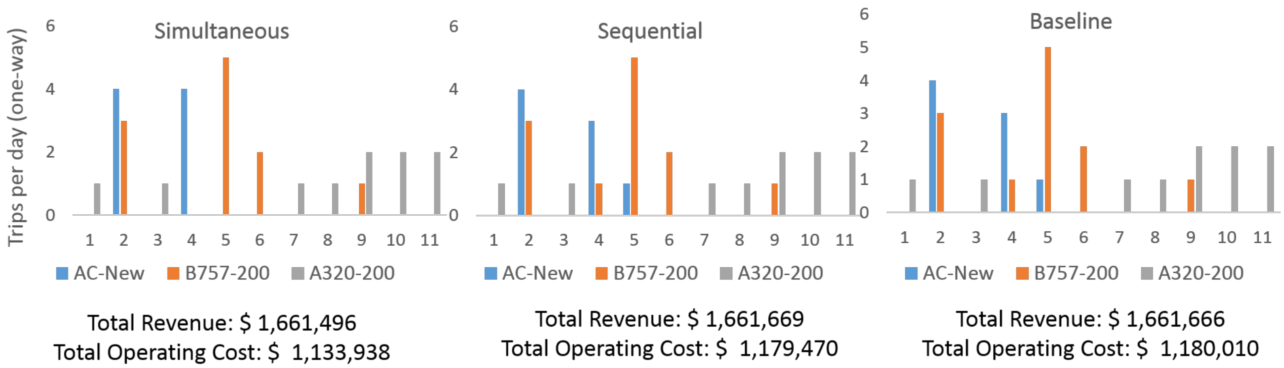


**Fig. 8 Results showing allocation of different aircraft types for the three scenarios together with the fleet-level revenue and cost.**

sequential) does result in higher revenue. However, the airline incurs additional costs for making these extra trips. The baseline and the sequential approach generate more revenue but also incurs an additional cost at the same time.

Although it was not apparent from the Table 3 how the new aircraft contributes towards the improvement in profit for the simultaneous scenario, further investigation revealed that the new aircraft optimized using the simultaneous approach is more economical to operate on the routes where allocated, compared to the other two cases. Figure 9 below shows the average operating cost per trip for the new aircraft across the three scenarios. The "hand-off" between how the aircraft is designed and how it is operated by the airline becomes obvious here. Although the aircraft optimized using the sequential approach yields the lowest operating cost at the design mission range (see Table 3), the aircraft designed in the sequential scenario only has a marginal reduction in average operating cost per trip (about 0.35%) as appears in Fig. 9. On the other hand, the simultaneous approach at the time of optimizing the aircraft takes into account how the airline allocates this new aircraft (also how the ticket prices are set by the revenue management subspace) and accordingly, the approach yields an aircraft design solution that has reduced average operating cost per trip (about 8.8%) on the routes where allocated, which, in turn, contributes to the improvement in the airline profit.

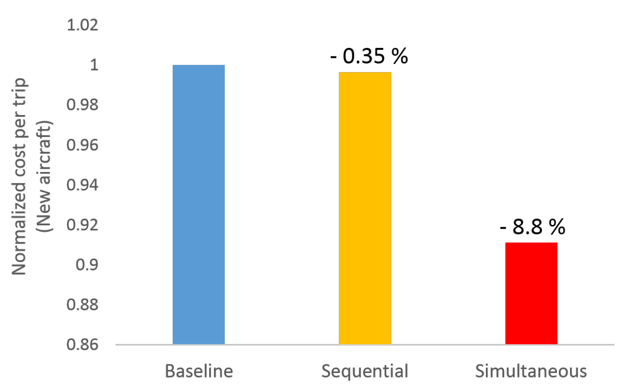


**Fig. 9 Comparison of the normalized average operating cost per trip for the new aircraft**

### A. Coupling sensitivity analysis and global exploring capability of AMIEGO

The studies in the previous section demonstrate the benefits of capturing the interactions between the subspaces of aircraft design, airline allocation, and revenue management. In this section, the authors investigate which subspace contributes the most towards improving the fleet-level profit of the airline. Considering the baseline (or even the sequential) allocation solution and solving only the aircraft design and revenue management subspaces simultaneously (i.e., the airline allocation result is kept fixed at the baseline solution, and the design and the revenue management variables are made to vary simultaneously), there is only a marginal increase in the airline profit. Figure 10 compares the profit using the three scenarios while keeping allocation solution of the simultaneous approach fixed at the baseline case.

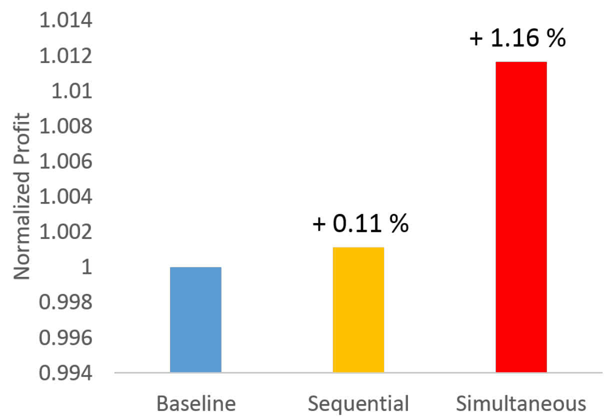


**Fig. 10 Normalized profit comparison with the allocation variables kept fixed at the baseline solution (not to scale)**

Figure 10 reveals that the allocation subspace plays an important role towards the airline profit improvement for the simultaneous approach. This, in turn, attests to the global exploring capability of AMIEGO to identify the right set of integer type allocation variables that improve the profit drastically over the other approaches to this problem (as seen in Fig. 6). Without AMIEGO, merely performing a simultaneous optimization of the design and revenue management subspace (both have continuous type decision variables) starting from the baseline or the sequential allocation result does not lead to a very good improvement in the airline profit as evident from the Fig. 10.

However, the AMIEGO framework does not guarantee global convergence( see Fig. 7). For routes 1 and 3, the airline takes a loss in all three scenarios. Using the simultaneous approach, one would expect the airline not to fly those two routes and improve the profit further. However, the improvement in profit for not flying those routes would have been only 2.34% over the already improved profit (compared to sequential and baseline case), and AMIEGO overlooks this enhancement. The framework is expected to lead to a near-global solution with as low computational cost as possible. The results discussed in this paper do show that, for the DARM problem of interest here, AMIEGO is able to find good –but not globally-optimal-solutions to the mixed integer nonlinear programming problem, and does so with acceptable computational cost.

### B. Addressing Computational Cost

AMIEGO solves the above DARM problem in as few as 4 expensive continuous runs[‡] (Step 2 of AMIEGO). Given the design-allocation-revenue management problem has 33 integer variables, the ability to find a high-quality answer (if not globally optimal) after using only 4 expensive continuous optimizations is very competitive for an MINLP solver. AMIEGO brings down the computational cost drastically by exploiting the formulation of the DARM problem. This exploitation of the mathematical structure is an ad-hoc implementation, specific to this DARM problem, to significantly reduce the computational cost. A generalized implementation of this approach will be considered as a part of future work.

Revisiting Eq. 2 and Eq. 3, –the allocation constraints; the demand constraints (see below)

$$\sum_{a=1}^{A}(trip_{a,j} \cdot pax_{a,j}) \leq dem_j, \forall j = 1, .., J$$

can always be satisfied for any values of $trip_{a,j}$. This is because the limiting value of the variable $pax_{a,j} = 0$ will always satisfy the demand constraints irrespective of the value of $trip_{a,j}$. However, this is not the case for the aircraft

[‡] A smaller penalty multiplier factor increases the total number of AMIEGO iterations.

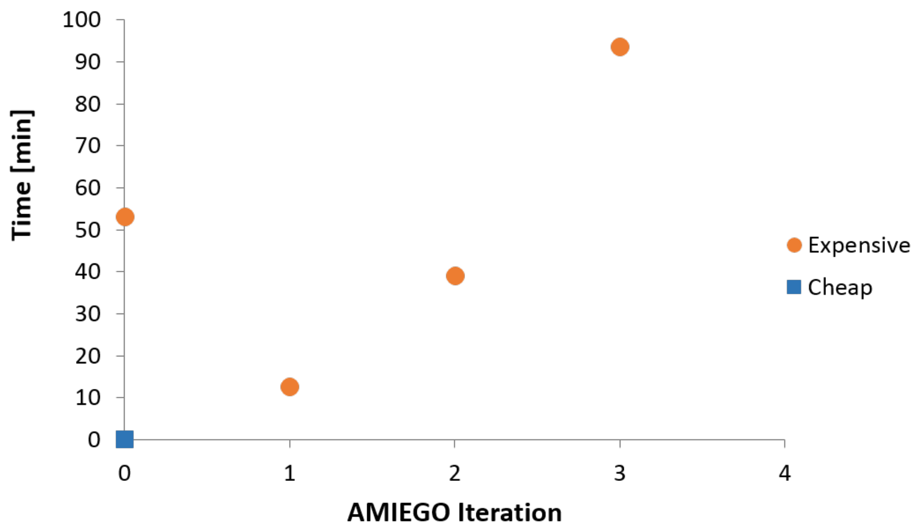


**Fig. 11 The total number of continuous optimization runs (both expensive and cheap) by AMIEGO (Step2).**

count constraints (Eq. 3), also reappear below

$$2\sum_{j=1}^{J} trip_{a,j} \cdot [BH_{a,j}(pax_{a,j}) + MH_{a,j} + 1] \leq 24 \cdot fleet_a, \forall a = 1, .., A$$

If the value of block hours of an aircraft type on each route is assumed to be independent of the number of passengers carried, which is an assumption (there is not much deviation in block hours as a function of the number of passengers carried), then this is a linear function of only the integer valued variables $trip_{a,j}$. The resulting equation is very fast to evaluate for any given value of $\mathbf{x_I}$ (or $trip_{a,j}$). Then, any point $\mathbf{x_I}$, that does not satisfy the count constraints need not go to step 2 of AMIEGO for the continuous optimization runs. Because the point is infeasible with respect to the count constraints, sending it for continuous optimization will be a waste of computational effort, because changing the continuous variables will not lead to satisfying this constraint. This implementation can be seen as a multi-fidelity-like approach where, if the feasibility check fails, the outcome of the expensive continuous optimization are evaluated using an inexpensive tool. This ad-hoc implementation within the AMIEGO framework for this specific problem appears in Fig. 12.

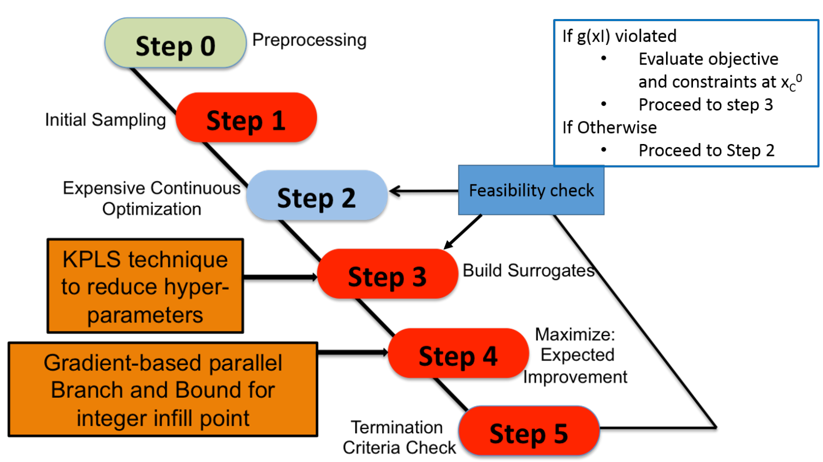


**Fig. 12 AMIEGO framework with the pre-optimization feasibility check**

This simple check, right before step 2 in AMIEGO, saves enormous computational effort (66 inexpensive runs (blue squares) stacked on top of each other in Fig. 11). However, for these pre-checked infeasible points, AMIEGO still needs the objective function value $f_{opt}$ (airline profit) and the constraint values $g(\mathbf{x_I}, \mathbf{x_C})$. These values of the objective and constraints are evaluated at $\mathbf{x}_C^0$ (the default starting point for the continuous optimization). Because the majority of the starting initial set of integer points belongs to the infeasible design space, a crude estimate is sufficient for this application. These infeasible points penalizes the expected improvement function via the exterior penalty approach. To AMIEGO, it appears as if these points came from the expensive continuous optimization runs in step 2.

From Figure 11, it appears that 66 out of the 67 initial integer points only required a few seconds for the continuous optimization runs. This shows that these are all points that failed the feasibility pre-check and the remaining efforts for this iteration uses the quick estimation to compute the objective and the constraints. The only expensive run at the start is the baseline solution that replaces one of the $2k_I + 1$ points from Latin Hypercube Sampling because AMIEGO needs at least one feasible point to start with. After that, AMIEGO performs 3 more expensive continuous optimization runs and finally terminates when it expects no further improvement in the surrogate modeling. The idea behind this implementation is to represent the vast majority of the infeasible region using a fast approximation technique and model the feasible region with the actual expensive run results, thereby saving a significant amount of expensive continuous optimization runs where they would not significantly contribute to the search.

# V. Conclusion and future work

Considering the design of a new aircraft together with the operations and economics of the airline is a challenging task. The effort here seeks to identify the design requirements of a new aircraft yet-to-be-acquired by the airline. To address this challenge we develop a process that considers the operational and economic characteristics of the airline as decision variables at the time of performing design optimization of the new aircraft. This consideration of operational and economics factors at the time of designing the new aircraft leads to a tightly coupled problem formulation which improves the fleet-level objective of the airline. However, the combined approach leads to a Mixed Integer Nonlinear Programming problem formulation and is difficult to solve. Leveraging the recently developed optimization framework AMIEGO along with the computational framework and the parallel computing tools, the problem now is computationally tractable.

The results reveal the presence of "synergism" between the subspaces. One may not be able to analyze the whole problem starting from a single subspace for these type of problems. Every subspace works in unison to contribute towards improving the airline-level objective. The simultaneous approach performs better than its sequential decomposition counterpart due to the fact that in the simultaneous approach, AMIEGO finds for an aircraft design, a fleet allocation, and revenue management ticket pricing that maximize airline profit. In the sequential approach, the aircraft is first sized to minimize total operating cost for its design mission range with no information how this new aircraft will be operated by the airline. Then all the aircraft (new and existing) are allocated in the allocation subspace, with no information about what ticket price the revenue management will set and how much demand this ticket price will attract. In the simultaneous approach, the aircraft design affects airline operational and economics, which in turn affects the design variables in the aircraft sizing subspace.

The demonstrations of how AMIEGO solves the combined new aircraft design –airline allocation –revenue management problem uses a low fidelity representation of all three subspaces. This low level of fidelity allowed demonstration of AMIEGO's efficacy at solving the DARM problem and of the potential improvement available when capturing the coupling amongst design, allocation, and revenue management. However, the usefulness of the results obtained in this effort is limited by the analyses and models. The current application problem used FLOPS as the aircraft sizing tool and employed a simplistic representation of a typical airline operation. The effort to replace FLOPS with a high-fidelity aircraft design tool is underway [6, 17], and we will then solve the DARM problem with high-fidelity aircraft design analyses using AMIEGO within NASA's OpenMDAO framework.

# VI. Expected Significance

The expected significance of this work is to deliver a novel engineering design process that accounts for the fleet-level objective of an airline that wishes to acquire a new aircraft type in a mix of existing fleet, taking into account its operational and economic behavior. The conventional multi-point aircraft design optimization does not capture the true operational characteristics as how the aircraft is being used by the airlines. The proposed framework would help identify the next generation aircraft design solution that when included by the airlines in their mix of existing fleet of aircraft, would enable them to improve their fleet-level objectives.

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